Collision

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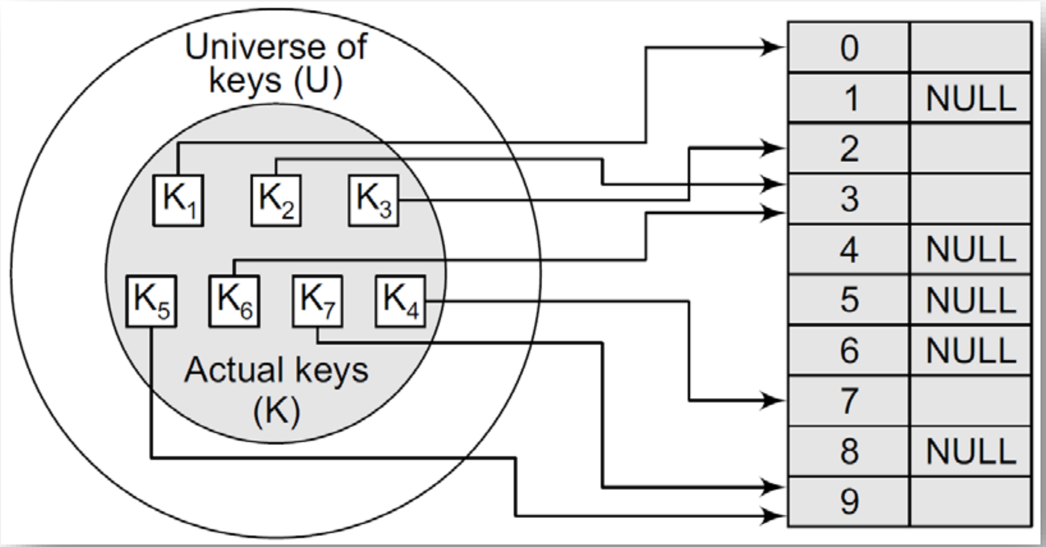
Review

- Hash table is a data structure in which keys are mapped to array positions by a **hash function**



  - Division method

    $h(x) = x \bmod M$

  - Multiplication method

    $h(x) = \lfloor m(xA \bmod 1) \rfloor$

  - Mid-square method

    $h(x) = r - digit(x^2)$

  - Folding method

    $h(x) = r - digit\left(sum\left(divide(x)\right)\right)$

Collision

- When two or more keys map to the same memory location, a **collision** is said to occur

- A method used to solve the problem of collision, also called **collision resolution technique**, is applied
  - Open addressing
  - Chaining

Open Addressing

- By using the technique, the hash table contains two types of values: **sentinel values** (e.g., –1) and **data values**
  - The sentinel value indicates that the location contains no data value at present but can be used to hold a value

- If the location already has some data value stored in it, then other slots are examined systematically in the forward direction to find a free slot
  - If even a single free location is not found, then we have an OVERFLOW condition

- The process of examining memory locations in the hash table is called **probing**
  - linear probing, quadratic probing, double hashing, and rehashing

Linear Probing

- The simplest approach to resolve a collision is linear probing
  - An extension of the division method

- If a value is already stored at a location generated by $h(x)$, then the following hash function is used to resolve the collision

$$h(x, i) = [h'(x) + i] \bmod M$$

  - M is the size of the hash table, $h'(x) = x \bmod M$, and i is the probe number that varies from 0 to $M-1$

  - When we have to store a value, we try the slots: $[h'(x)] \bmod M$, $[h'(x) + 1] \bmod M$, $[h'(x) + 2] \bmod M$, $[h'(x) + 3] \bmod M$, and so no, until a vacant location is found

$h(x, i) = [h'(x) + i] \bmod M$

Example.

- Consider a hash table of size 10, please use linear probing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table
  - Initial

0	1	2	3	4	5	6	7	8	9
–1	–1	–1	–1	–1	–1	–1	–1	–1	–1

  - Step 1
    - $x = 72$
    - $h(72,0) = [h'(72) + 0] \bmod 10$
      $= [(72 \bmod 10) + 0] \bmod 10 = 2$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	–1	–1	–1	–1

$h(x,i) = [h'(x) + i] \bmod M$

Example..

- Consider a hash table of size 10, please use linear probing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	–1	–1	–1	–1

  - Step 2
    - $x = 27$
    - $h(27,0) = [h'(27) + 0] \bmod 10$
      $= [(27 \bmod 10) + 0] \bmod 10 = 7$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	–1	27	–1	–1

  - Step 3
    - $x = 36$
    - $h(36,0) = [h'(36) + 0] \bmod 10$
      $= [(36 \bmod 10) + 0] \bmod 10 = 6$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	36	27	–1	–1

Example...

$h(x,i) = [h'(x) + i] \bmod M$

- Consider a hash table of size 10, please use linear probing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	36	27	–1	–1

  – Step 4
    - $x = 24$
    - $h(24,0) = [h'(24) + 0] \bmod 10$
      $= [(24 \bmod 10) + 0] \bmod 10 = 4$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	24	–1	36	27	–1	–1

  – Step 5
    - $x = 63$
    - $h(63,0) = [h'(63) + 0] \bmod 10$
      $= [(63 \bmod 10) + 0] \bmod 10 = 3$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	63	24	–1	36	27	–1	–1

$h(x, i) = [h'(x) + i] \bmod M$

Example….

- Consider a hash table of size 10, please use linear probing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table
  - Step 6

0	1	2	3	4	5	6	7	8	9
0	81	72	63	24	–1	36	27	–1	–1

    - $x = 81$
    - $h(81,0) = [h'(81) + 0] \bmod 10$
      $= [(81 \bmod 10) + 0] \bmod 10 = 1$
  - Step 7

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	92	36	27	–1	–1

    - $x = 92$
    - $h(92,0) = [h'(92) + 0] \bmod 10$
      $= [(92 \bmod 10) + 0] \bmod 10 = 2$
    - $h(92,1) = [h'(92) + 1] \bmod 10$
      $= [(92 \bmod 10) + 1] \bmod 10 = 3$
    - $h(92,2) = [h'(92) + 2] \bmod 10$
      $= [(92 \bmod 10) + 2] \bmod 10 = 4$
    - $h(92,3) = [h'(92) + 3] \bmod 10$
      $= [(92 \bmod 10) + 3] \bmod 10 = 5$

$h(x,i) = [h'(x) + i] \bmod M$

Example.....

- Consider a hash table of size 10, please use linear probing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	92	36	27	–1	–1

  - Step 8
    - $x = 101$
    - $h(101,0) = [h'(101) + 0] \bmod 10$
      $= [(101 \bmod 10) + 0] \bmod 10 = 1$
    - $h(101,1) = [h'(101) + 1] \bmod 10$
      $= [(101 \bmod 10) + 1] \bmod 10 = 2$
    - …
    - $h(101,7) = [h'(101) + 7] \bmod 10$
      $= [(101 \bmod 10) + 7] \bmod 10 = 8$

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	92	36	27	101	–1

Quadratic Probing

- If a value is already stored at a location generated by $h(x)$, then the following hash function is used to resolve the collision

$$h(x, i) = [h'(x) + c_1 \times i + c_2 \times i^2] \bmod M$$

  - M is the size of the hash table, $h'(x) = x \bmod M$, i is the probe number that varies from 0 to $M - 1$, and c_1 and c_2 are constants such that $c_1 \neq 0$ and $c_2 \neq 0$

$h(x, i) = [h'(x) + c_1 \times i + c_2 \times i^2] \bmod M$

Example.

- Consider a hash table of size 10, please use quadratic probing to insert the keys 72, 27, 36, 24, 63, 81, 101, and 92 into the table
  - If we set $c_1 = 1$ and $c_2 = 3$
  - Initial

0	1	2	3	4	5	6	7	8	9
–1	–1	–1	–1	–1	–1	–1	–1	–1	–1

  - Step 1
    - $x = 72$
    - $h(72,0) = [h'(72) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(72 \bmod 10) + 0 + 0] \bmod 10 = 2$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	–1	–1	–1	–1

Example..

- Consider a hash table of size 10, please use quadratic probing to insert the keys 72, 27, 36, 24, 63, 81, 101, and 92 into the table
  - Step 2
    - $x = 27$
    - $h(27,0) = [h'(27) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(27 \bmod 10) + 0 + 0] \bmod 10 = 7$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	–1	27	–1	–1

  - Step 3
    - $x = 36$
    - $h(36,0) = [h'(36) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(36 \bmod 10) + 0 + 0] \bmod 10 = 6$

0	1	2	3	4	5	6	7	8	9
–1	–1	72	–1	–1	–1	36	27	–1	–1

$h(x,i) = [h'(x) + c_1 \times i + c_2 \times i^2] \bmod M$

Example…

- Consider a hash table of size 10, please use quadratic probing to insert the keys 72, 27, 36, 24, 63, 81, 101, and 92 into the table
  - Step 4
    - $h(24,0) = [h'(24) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(24 \bmod 10) + 0 + 0] \bmod 10 = 4$
  - Step 5
    - $h(63,0) = [h'(63) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(63 \bmod 10) + 0 + 0] \bmod 10 = 3$
  - Step 6
    - $h(81,0) = [h'(81) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(81 \bmod 10) + 0 + 0] \bmod 10 = 1$

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	–1	36	27	–1	–1

Example….

- Consider a hash table of size 10, please use quadratic probing to insert the keys 72, 27, 36, 24, 63, 81, 101, and 92 into the table

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	–1	36	27	–1	–1

  – Step 7
    - $h(101,0) = [h'(101) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(101 \bmod 10) + 0 + 0] \bmod 10 = 1$
    - $h(101,1) = [h'(101) + 1 \times 1 + 3 \times 1^2] \bmod 10$
      $= [(101 \bmod 10) + 1 + 3] \bmod 10 = 5$

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	101	36	27	–1	–1

$h(x, i) = [h'(x) + c_1 \times i + c_2 \times i^2] \bmod M$

Example.....

- Consider a hash table of size 10, please use quadratic probing to insert the keys 72, 27, 36, 24, 63, 81, 101, and 92 into the table

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	101	36	27	–1	–1

  - Step 8
    - $h(92,0) = [h'(92) + 1 \times 0 + 3 \times 0^2] \bmod 10$
      $= [(92 \bmod 10) + 0 + 0] \bmod 10 = 2$
    - $h(92,1) = [h'(92) + 1 \times 1 + 3 \times 1^2] \bmod 10$
      $= [(92 \bmod 10) + 1 + 3] \bmod 10 = 6$
    - $h(92,2) = [h'(92) + 1 \times 2 + 3 \times 2^2] \bmod 10$
      $= [(92 \bmod 10) + 2 + 12] \bmod 10 = 6$
    - $h(92,3) = [h'(92) + 1 \times 3 + 3 \times 3^2] \bmod 10$
      $= [(92 \bmod 10) + 3 + 27] \bmod 10 = 2$
    - $h(92,4) = [h'(92) + 1 \times 4 + 3 \times 4^2] \bmod 10$
      $= [(92 \bmod 10) + 4 + 48] \bmod 10 = 4$
    - $h(92,5) = [h'(92) + 1 \times 5 + 3 \times 5^2] \bmod 10$
      $= [(92 \bmod 10) + 5 + 75] \bmod 10 = 2$

$h(x, i) = [h'(x) + c_1 \times i + c_2 \times i^2] \bmod M$

Example……

- Consider a hash table of size 10, please use quadratic probing to insert the keys 72, 27, 36, 24, 63, 81, 101, and 92 into the table
  - $h(92,6) = [h'(92) + 1 \times 6 + 3 \times 6^2] \bmod 10$
    $= [(92 \bmod 10) + 6 + 108] \bmod 10 = 6$
  - $h(92,7) = [h'(92) + 1 \times 7 + 3 \times 7^2] \bmod 10$
    $= [(92 \bmod 10) + 7 + 147] \bmod 10 = 6$
  - $h(92,8) = [h'(92) + 1 \times 8 + 3 \times 8^2] \bmod 10$
    $= [(92 \bmod 10) + 8 + 192] \bmod 10 = 2$
  - $h(92,9) = [h'(92) + 1 \times 9 + 3 \times 9^2] \bmod 10$
    $= [(92 \bmod 10) + 9 + 243] \bmod 10 = 4$

  – One of the major **drawbacks** of quadratic probing is that a sequence of successive probes **may only explore a fraction of the table**, and **this fraction may be quite small**
    - If this happens, then we will not be able to find an empty location in the table despite the fact that the table is not full

Double Hashing

- Double hashing uses one hash value and then repeatedly steps forward an interval until an empty location is reached
  - The interval is decided using a second, independent hash function, hence the name **double hashing**

$$h(x, i) = [h_1(x) + i \times h_2(x)] \bmod M$$

  - M is the size of the hash table
  - $h_1(x)$ and $h_2(x)$ are two hash functions
    - $h_1(x) = x \bmod M$
    - $h_2(x) = x \bmod M'$
    - M' is chosen to be less than M

      We can choose $M' = M - 1$ or $M' = M - 2$
  - i is the probe number that varies from 0 to $M-1$

Example.

- Consider a hash table of size 10, please use double hashing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table
  - If we set $h_1 = x \bmod 10$ and $h_2 = x \bmod 8$
  - Initial

0	1	2	3	4	5	6	7	8	9
–1	–1	–1	–1	–1	–1	–1	–1	–1	–1

  - Step 1
    - $x = 72$
    - $h(72,0) = [h_1(72) + 0 \times h_2(72)] \bmod 10$
      $= [(72 \bmod 10) + 0 \times (72 \bmod 8)] \bmod 10 = 2$
  - Step 2
    - $x = 27$
    - $h(27,0) = [h_1(27) + 0 \times h_2(27)] \bmod 10$
      $= [(27 \bmod 10) + 0 \times (27 \bmod 8)] \bmod 10 = 7$

Example..

- Consider a hash table of size 10, please use double hashing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table
  - Step 3
    - $x = 36$
    - $h(36,0) = [h_1(36) + 0 \times h_2(36)] \bmod 10$
      $= [(36 \bmod 10) + 0 \times (36 \bmod 8)] \bmod 10 = 6$
  - Step 4
    - $x = 24$
    - $h(24,0) = [h_1(24) + 0 \times h_2(24)] \bmod 10$
      $= [(24 \bmod 10) + 0 \times (24 \bmod 8)] \bmod 10 = 4$
  - Step 5
    - $x = 63$
    - $h(63,0) = [h_1(63) + 0 \times h_2(63)] \bmod 10$
      $= [(63 \bmod 10) + 0 \times (63 \bmod 8)] \bmod 10 = 3$

Example...

- Consider a hash table of size 10, please use double hashing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table
  - Step 6
    - $x = 81$
    - $h(81{,}0) = [h_1(81) + 0 \times h_2(81)] \bmod 10$
      $= [(81 \bmod 10) + 0 \times (81 \bmod 8)] \bmod 10 = 1$

0	1	2	3	4	5	6	7	8	9
–1	81	72	63	24	–1	36	27	–1	–1

  - Step 7
    - $x = 92$
    - $h(92{,}0) = [h_1(92) + 0 \times h_2(92)] \bmod 10$
      $= [(92 \bmod 10) + 0 \times (92 \bmod 8)] \bmod 10 = 2$
    - $h(92{,}1) = [h_1(92) + 1 \times h_2(92)] \bmod 10$
      $= [(92 \bmod 10) + 1 \times (92 \bmod 8)] \bmod 10 = 6$
    - $h(92{,}2) = [h_1(92) + 2 \times h_2(92)] \bmod 10$
      $= [(92 \bmod 10) + 2 \times (92 \bmod 8)] \bmod 10 = 0$

$h(x, i) = [h_1(x) + i \times h_2(x)] \bmod M$

Example....

- Consider a hash table of size 10, please use double hashing to insert the keys 72, 27, 36, 24, 63, 81, 92, and 101 into the table

0	1	2	3	4	5	6	7	8	9
92	81	72	63	24	–1	36	27	–1	–1

  - Step 8
    - $x = 101$
    - $h(101,0) = [h_1(101) + 0 \times h_2(101)] \bmod 10$
      $= [(101 \bmod 10) + 0 \times (101 \bmod 8)] \bmod 10 = 1$
    - $h(101,1) = [h_1(101) + 1 \times h_2(101)] \bmod 10$
      $= [(101 \bmod 10) + 1 \times (101 \bmod 8)] \bmod 10 = 6$
    - $h(101,2) = [h_1(101) + 2 \times h_2(101)] \bmod 10$
      $= [(101 \bmod 10) + 2 \times (101 \bmod 8)] \bmod 10 = 1$
    - ....

Rehashing

- When the hash table becomes nearly full, the number of collisions increases, thereby degrading the performance of insertion and search operations
  - A better option is to **create a new hash table** with size double of the original hash table

- By performing the rehashing, all the entries in the original hash table will then have to be moved to the new hash table
  - This is done by taking each entry, computing its new hash value, and then inserting it in the new hash table

- Though rehashing seems to be a simple process, it is **quite expensive** and must therefore not be done frequently

Example

- Consider the hash table of size 5 given below
  - The hash function used is $h(x) = x \bmod 5$ with linear probing

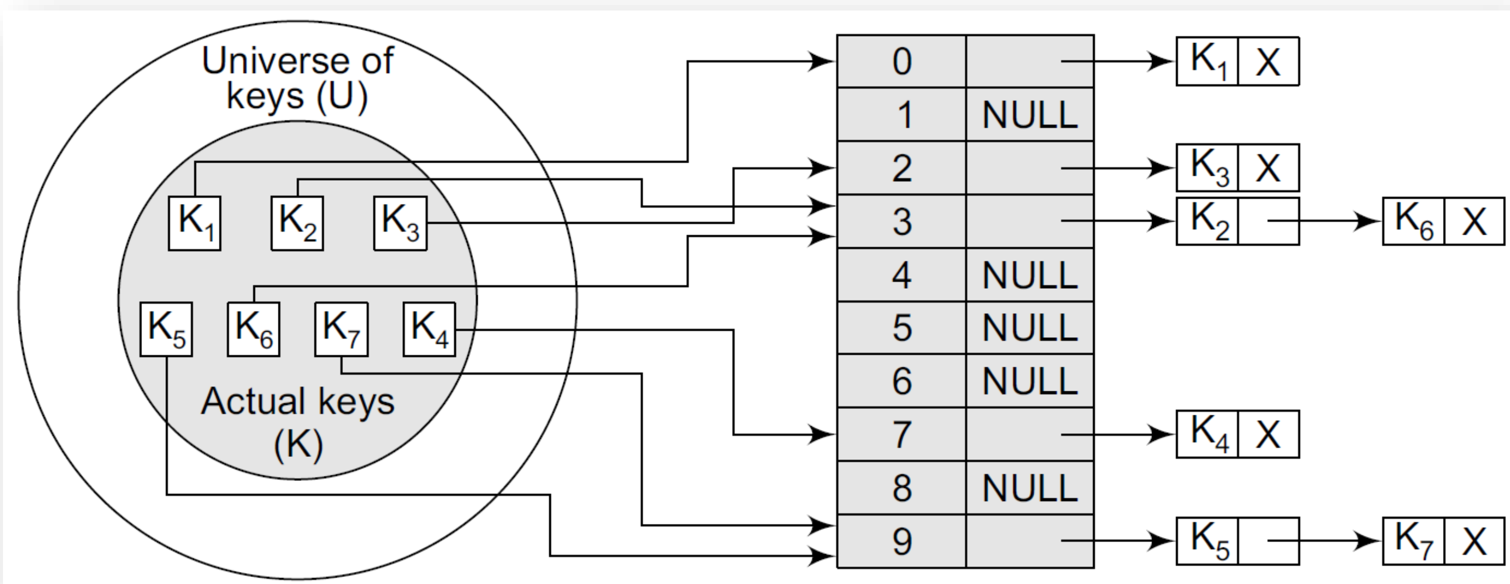
0	1	2	3	4
	26	31	43	17

  - Rehash the entries into to a new hash table
    - Note that the new hash table is of 10 locations, double the size of the original table
    - Rehash the key values from the old hash table into the new one using hash function $h(x) = x \bmod 10$ with linear probing

0	1	2	3	4	5	6	7	8	9
	31		43			26	17		

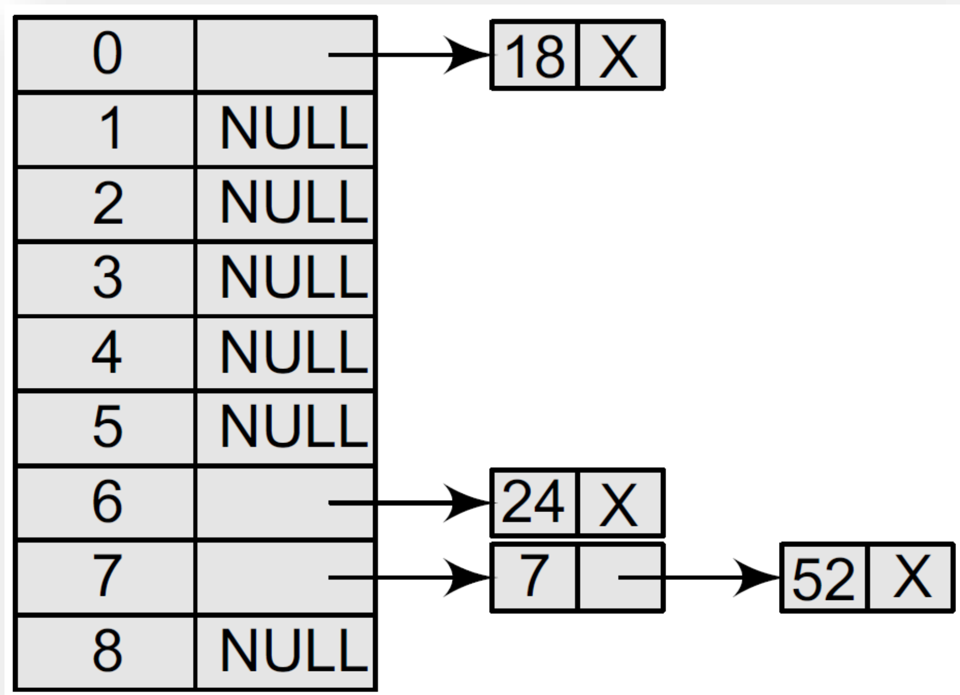
Chaining

- In chaining, each location in a hash table stores a pointer to a linked list that contains all the key values that were hashed to that location
  - Location n in the hash table points to the head of the linked list of all the key values that hashed to n
  - If no key value hashes to n, then location n in the hash table contains NULL



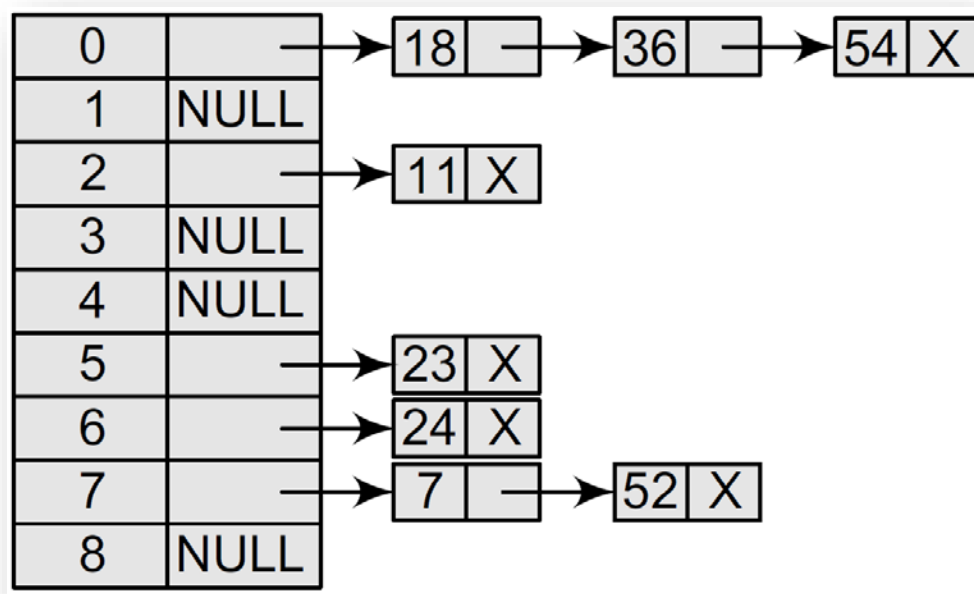
Example.

- Insert the keys 7, 24, 18, 52, 36, 54, 11, and 23 in a chained hash table of 9 memory locations
  - The hash function is $h(x) = x \bmod 9$
  - Step 1
    - $x = 7$
    - $h(7) = 7 \bmod 9 = 7$
  - Step 2
    - $x = 24$
    - $h(24) = 24 \bmod 9 = 6$
  - Step 3
    - $x = 18$
    - $h(18) = 18 \bmod 9 = 0$
  - Step 4
    - $x = 52$
    - $h(52) = 52 \bmod 9 = 7$



Example..

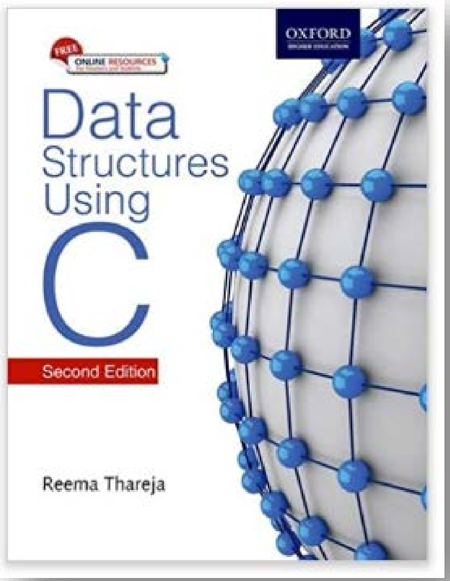
- Insert the keys 7, 24, 18, 52, 36, 54, 11, and 23 in a chained hash table of 9 memory locations
  - Step 5
    - $x = 36$
    - $h(36) = 36 \bmod 9 = 0$
  - Step 6
    - $x = 54$
    - $h(54) = 54 \bmod 9 = 0$
  - Step 7
    - $x = 11$
    - $h(11) = 11 \bmod 9 = 2$
  - Step 8
    - $x = 23$
    - $h(23) = 23 \bmod 9 = 5$



Summary

9/14	Course Overview
9/16	Program, Algorithm & Recursion
9/21	Performance Analysis
9/23	Advanced Analysis
9/28	Arrays
9/30	Stacks
10/5	Infix, Prefix and Postfix
10/7	Queues
10/12	Trees
10/14	Traversal for Binary Tree
10/19	Binary Search Trees
10/21	AVL Trees
10/26	Red-Black Trees
10/28	Splay Trees
11/02	B Trees
11/04	B+, 2-3, Huffman Trees
11/09	Searching

11/23	Bubble, Insertion & Tree Sorts
11/25	Selection, Merge & Radix Sorts
11/30	Quick, Heap and Shell Sorts
12/02	Heap & Its Variants
12/07	Undirected & Directed Graphs
12/09	Advanced Graphs
12/14	Minimum Spanning Trees
12/16	Shortest Path Algorithms
12/21	Graph Traversal Algorithms
12/23	Hash Functions
12/	Collision



Grading

- Homework: 55%
  - HW0: 5%
  - HW1~5: 10%
- Midterm: 25%
- Final: 30%

Questions?

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